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On Naturalness of Scalar Fields and the Standard Model

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We discuss how naturalness predicts the scale of new physics. Two conditions on the scale are considered. The first is the more conservative condition due to Veltman (Acta Phys. Polon. B **12**, 437 (1981)). It requires that radiative corrections to the electroweak mass scale would be reasonably small. The second is the condition due to Barbieri and Giudice (Nucl. Phys. B **306**, 63 (1988)), which is more popular lately. It requires that physical mass scale would not be oversensitive to the values of the input parameters. We show here that the above two conditions behave differently if higher order corrections are taken into account. Veltman's condition is robust (insensitive to higher order corrections), while Barbieri-Giudice condition changes qualitatively. We conclude that higher order perturbative corrections take care of the fine tuning problem, and, in this respect, scalar field is a natural system. We apply the Barbieri-Giudice condition with higher order corrections taken into account to the Standard Model, and obtain new restrictions on the Higgs boson mass.

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It was pointed out in [1, 2, 3] that theories with scalar fields are facing a serious problem (and the Standard Model is among these). It consists in absence of a natural explanation for small values of masses of scalar particles. ("Small" here means much smaller than the possible fundamental scales like Plank mass or a unification scale.)

The problem appears as follows. Let us try to expand the physical mass in a series of bare couplings. In the one-loop approximation we have

$$m^2 = m_0^2 + \Lambda^2 P(\lambda_0, g). \tag{1}$$

Here m^2 is the squared mass of a scalar particle, m_0^2 is the corresponding bare mass of the fundamental Lagrangian of the model defined at the fundamental scale Λ, which is also used as a cutoff in the Feynman integrals, $P(\lambda_0, g)$ is a polynomial of dimensionless bare scalar field selfcoupling λ_0 and the rest of dimensionless bare couplings g of the model, and we neglected the corrections depending logarithmically on the cutoff. (For example, in the Standard Model, $P(\lambda_0, g) = 3(3g_2^2 + g_1^2 + 2\lambda_0 - 4y_t^2)/(32\pi^2)$, where g_1, g_2, and y_t are the gauge couplings of the gauge groups $SU(1)$, $SU(2)$, and top quark Yukawa coupling, respectively [5].) Here comes the question: How to keep m much less than Λ? One obvious option is to fine tune the values of m_0^2 and $P(\lambda_0, g)$ to make the two terms in the right-hand-side of Eq. (1) cancel against each other. But this seems not to be a natural way (thus the name of the problem—the naturalness problem). Another way is to ask for a model where $P(\lambda_0, g)$ is exactly zero (which is the case for softly broken supersymmetry models [4]). More generally, if one rejects unnatural fine tunings of fundamental parameters, introducing scalar fields one should also point out a mechanism that keeps the hierarchy between m and Λ (the hierarchy problem).

On a more practical note, Eq. (1) had been used [5, 6] to obtain the scale of new physics. The idea is not to consider Λ as a fundamental scale, but as a scale up to which we can use the low energy effective theory implying Eq. (1). One may restrict Λ requiring, for example [5], that the radiative correction to the mass squared would not exceed the bare mass squared:

$$|m^2 - m_0^2| < m_0^2. \tag{2}$$

In what follows we call this condition Veltman's condition.

Another possibility is to restrict not the magnitude of the radiative correction, but the sensitivity of the physical mass to small changes in the values of the bare couplings [6]:

$$\Big|\frac{\lambda_0}{m^2}\frac{\partial m^2}{\partial \lambda_0}\Big| < q, \tag{3}$$

where q parameterizes the strictness of our requirements (the value $q = 10$ was suggested in [6]). Hereafter, we call this condition the Barbieri-Giudice condition.

Now, assuming that the radiative correction to mass squared is positive ($P(\lambda_0, g) > 0$) and neglecting the difference between bare and physical couplings, Veltman's condition (2) implies the following restriction on Λ:

$$\Lambda^2 < \frac{m^2}{2P(\lambda, g)}, \tag{4}$$

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where λ denotes the physical coupling corresponding to the bare coupling λ_0. The quantities in the right-hand-side of this inequality are measurable. So we can substitute the measured values, and obtain an estimate for the scale of new physics. This program was realized in Ref. [5] for the Standard Model. The outcome is that the scale for the new physics is estimated by 1.2 TeV. Similarly, if we assume Eq. (1), Barbieri-Giudice condition (3) implies

$$\Lambda^2 < q \frac{m^2}{\lambda P'(\lambda, g)}, \tag{5}$$

where the prime over P denotes derivative with respect to λ.

As we see, the two conditions yield similar upper bounds for the scale of new physics. In fact, Veltman's condition and Barbieri-Giudice condition are rather different, and the similarity of the bounds (4) and (5) is due to the use of the leading order formula (1).

Let us consider what may be the influence of higher order perturbative corrections on the bounds (4) and (5). This problem was briefly considered in Ref. [8]. It was observed that higher order corrections modify the polynomial $P(\lambda_0, g)$ from (1) (even making it dependent on Λ logarithmically). If this would be the only way higher order corrections are getting involved, they could not influence significantly the bounds (4) and (5) (at least, at small couplings).

Unfortunately, there are important higher order corrections overlooked in Ref. [8]: In higher orders of the expansion of the physical mass squared in powers of the bare couplings, Eq. (1), higher powers of Λ will appear, and the larger the order of perturbation theory, the larger is the power of Λ appearing in the right-hand-side of Eq. (1). For example, in the third order in λ_0 there is a diagram with two tadpoles attached to the scalar propagator. It gives contribution proportional to $\lambda_0^3 \Lambda^4 / m_0^2$. Similarly, in the expansion of the physical couplings in powers of bare couplings, infinitely high powers of Λ appear, and the power of Λ appearing in the expansion is bounded only if we consider a finite order of the perturbation theory in λ_0.

A direct approach is to study the powers of Λ appearing in the expansion of physical parameters in powers of bare couplings. This may be an interesting problem, but there is a shortcut allowing one to avoid it. Indeed, for renormalizable theories, dependence of *bare couplings* on the cutoff is known if they are expressed in terms of the *physical couplings* [7]. Let us reiterate: for renormalizable theory, *bare* mass squared of a scalar particle expressed as a series in powers of *physical* couplings with coefficients of the expansion depending on the cutoff, *physical* masses and renormalization scale grows not faster than the cutoff squared. Is this statement compatible with the appearance of higher powers of the cutoff in the right-hand-side of Eq. (1)? It is easy to check that there is no contradiction. Indeed, schematically, if we take the renormalization scale to be of the order of physical mass, the bare mass squared and the bare coupling are expressed as follows

$$m_0^2 = m^2 - \Lambda^2 P(\lambda, g), \tag{6}$$

$$\lambda_0 = \lambda + \log(\frac{\Lambda^2}{m^2}) \frac{\beta(\lambda, g)}{2}, \tag{7}$$

where $P(\lambda, g)$ is (in the leading order) the same polynomial as in Eq.(1), and $\beta(\lambda, g)$ is the leading order of the beta function governing the renormalization group evolution of coupling λ. If we use the above expressions as equations for m^2 and λ, we can determine the expansions of m^2 and λ in powers of λ_0. It is easy to check that both power series involve arbitrary high powers of the cutoff. The reason for the appearance of the high powers of Λ in the expansions is the presence of m^2 in the argument of the logarithm. (Logarithmic term is also present in the formula for bare mass, but we dropped it, because it is insignificant for further reasoning.)

If we put $\beta(\lambda, g) = 0$ in Eq. (7), we derive the bounds (4) and (5) from Veltman's condition (2) and Barbieri-Giudice condition (3), respectively. Evidently, the bound (4) is not influenced by nonzero $\beta(\lambda, g)$ in any way. In what follows, we see how the fact that $\beta(\lambda, g) \neq 0$ influences the bound (5).

We need to compute the derivative $\partial m^2 / \partial \lambda_0$ involved in Barbieri-Giudice condition (3). More generally, we need to compute the entries of the matrix

$$A = \begin{pmatrix} \frac{\partial \lambda}{\partial \lambda_0} & \frac{\partial \lambda}{\partial m_0^2} \\ \frac{\partial m^2}{\partial \lambda_0} & \frac{\partial m^2}{\partial m_0^2} \end{pmatrix}. \tag{8}$$

The inverse of the desired A can be computed with Eqs. (6) and (7):

$$A^{-1} \equiv B = \begin{pmatrix} \frac{\partial \lambda_0}{\partial \lambda} & \frac{\partial \lambda_0}{\partial m^2} \\ \frac{\partial m_0^2}{\partial \lambda} & \frac{\partial m_0^2}{\partial m^2} \end{pmatrix} \tag{9}$$

$$= \begin{pmatrix} 1 + \log(\frac{\Lambda^2}{m^2}) \frac{\beta'(\lambda, g)}{2} & -\frac{\beta(\lambda, g)}{2m^2} \\ -\Lambda^2 P'(\lambda, g) & 1 \end{pmatrix}, \tag{10}$$

where primes over β and P denote the derivative with respect to λ. Thus, the desired A is

$$A = \frac{1}{\det(B)} \begin{pmatrix} 1 & \frac{\beta(\lambda, g)}{2m^2} \\ \Lambda^2 P'(\lambda, g) & 1 + \log(\frac{\Lambda^2}{m^2}) \frac{\beta'(\lambda, g)}{2} \end{pmatrix}, \tag{11}$$

where

$$\det(B) = -\frac{\Lambda^2}{m^2} P'(\lambda, g) \frac{\beta(\lambda, g)}{2} + \log(\frac{\Lambda^2}{m^2}) \frac{\beta'(\lambda, g)}{2} + 1. \tag{12}$$

Now we see why it is important to keep nonzero $\beta(\lambda, g)$ in the consideration: Neglecting $\beta(\lambda, g)$ removes the most important first two terms in the right-hand-side of this expression. As a consequence, neglecting $\beta(\lambda, g)$ leads to a qualitative mistake in the estimate of the behavior of the matrix of derivatives A in the limit of large Λ.

Finally, in the limit of infinite Λ, we have:

$$A = \begin{pmatrix} 0 & 0 \\ -\frac{2m^2}{\beta(\lambda,g)} & 0 \end{pmatrix}. \tag{13}$$

Let us comment on Eq. (13). As we see, physical parameters—the observable mass and coupling—are not oversensitive to the values of the bare parameters defined at a large (e.g., fundamental) scale Λ. The leading order relation, Eq. (1), is misleading in this respect. In other words: Derivative of observable mass in bare coupling has a finite limit expressible in terms of observable parameters when the cutoff is removed. (This is the worst sensitivity we have: the physical coupling exhibits *universality*, i.e., it becomes independent of bare parameters at infinite cutoff; the physical mass becomes independent of the bare mass at infinite cutoff.) We conclude that the fine tuning problem is the problem of the leading order perturbative approximation, Eq. (1).

Now we can derive from the Barbieri-Giudice condition(3) the inequality

$$|\frac{2\lambda}{\beta(\lambda,g)}| < q, \tag{14}$$

where we neglected the difference between λ and λ_0.

Let us specialize inequality (14) to the case of the Standard Model. The Standard Model one-loop beta-function governing the evolution of the scalar selfcoupling λ is [9]

$$\begin{aligned}\beta(\lambda,g) &= \frac{6}{8\pi^2}(\lambda^2 - \lambda[\frac{1}{4}g_1^2 + \frac{3}{4}g_2^2 - g_t^2] \\ &+ \frac{1}{16}g_1^4 + \frac{1}{8}g_1^2g_2^2 + \frac{3}{16}g_2^4 - y_t^4), \end{aligned} \tag{15}$$

where g_1 and g_2 are gauge couplings of $SU(1)$ and $SU(2)$ respectively, $y_t = m_t/v$ (m_t is the mass of the top quark, and v is the vacuum expectation of the scalar field). The couplings involved in the expression for the beta function can be expressed via ratios of the masses and the scalar field vacuum expectation value v. In this way, for the Standard Model, Barbieri-Giudice condition (3) implies the following inequality:

$$\frac{4m_H^2v^2}{|p(m_H,m_Z,m_W,m_t)|} < \frac{3q}{4\pi^2}, \tag{16}$$

where $p(m_H,m_Z,m_W,m_t)$ is the following polynomial of the Higgs, Z, W and top quark masses:

$$\begin{aligned}p(m_H,m_Z,m_W,m_t) &= m_H^4 + m_H^2(2m_t^2 - m_Z^2 - 2m_W^2) \\ &- 4m_t^4 + m_Z^4 + 2m_W^4. \end{aligned} \tag{17}$$

Thus, Barbieri-Giudice condition (3) implies a restriction on the Higgs boson mass. Using known values, we see that inequality (16) forbids moderate values of the Higgs boson mass. For example, if we take $q = 10$, we obtain that the band of values of m_H approximately from 96 GeV to 540 GeV is forbidden. (The value for the upper boundary of the forbidden band is hardly reliable, because it corresponds to strongly interacting Higgs bosons.) If we relax the Barbieri-Giudice condition and take $q = 15$ (20), the forbidden band shrinks: it ranges from 113 (126) GeV to 438 (380) GeV.

Let us summarize our findings. Taking into account higher order perturbative corrections does not change the basic fact: radiative corrections to the electroweak scale are growing fast with cutoff. At 1.2 TeV the correction to the intermediate bosons mass squared is about a half of the total mass squared. Is it new physics that half of the observable mass scale is due to radiative corrections is a matter of convention. We consider such a situation as deserving the title of new physics. To say the least, perturbation theory looks jeopardized in such circumstances. Beyond perturbation theory, we still do not know any mechanism that would provide for small masses of the scalar particles.

On the other hand, if some unknown mechanism provides for small mass of scalar particles, perturbation theory is quite able to explain relative stability of the scalar mass against small variations in fundamental parameters. We demonstrated that there is no fine tuning problem in the theory of quantum scalar field, and derived inequality (16) in the Standard Model restricting the Higgs boson mass. Phenomenological consequences of this restriction will be studied elsewhere.

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